Solving Fuzzy Linear Programming Problem using Weighted Sum and Comparisons with Ranking Function

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Abstract— Multi-objective linear programming can be generated from fuzzy linear programming. This multi-objective linear programming can be further converted to single objective linear programming by using ranking function and weighted function. In this paper, it can be shown that the result of the single objective function which can be obtained by using ranking function matches with the result obtained by equal weight and unequal weight function. In this case it can be used for both triangular and trapezoidal fuzzy numbers.

Index Terms— Fully Fuzzy Linear Programming Problem, Triangular Fuzzy Number, Trapezoidal Fuzzy Number, Weighted Sum, Ranking Function.

1 Introduction

Linear programming (LP) is part of an important area of mathematics called "Optimization technique" as it is straightly used to find the most optimized solution to a given problem. It is also one of the simplest ways to perform optimization. A LP may be defined as the problem of maximizing or minimizing a linear function subject to linear constraints. The constraints may be equalities or inequalities. Working with linear programming model requires properly tuning the values of the parameters. Because the real world problems have a high level of complexity and the degree of uncertainty depends on many factors. In order to properly determine the value of these parameters, experts or decision makers needs to deal with this uncertainty and vagueness. Bellman and Zadeh [1] first proposed the concept of decision making in a fuzzy environment as a solution approach for this kind of problems. Zimmermann [2] presented an application of fuzzy optimization technique for multi-objective linear programming (MOLP) problems. Alanazi et al. [4] proposed a mathematical model using weighted sum method for multi-criteria decision making. A new method is discussed by Kumar et al. [3] for getting the optimal solution of fully fuzzy linear programming (FFLP) problems with equality constraints. Pandit [6] proposed a method to calculate the solution for multi-objective FFLP problems involving parameters represented by triangular fuzzy numbers. A new operation is proposed by Gani and Assarudeen [10] where triangular fuzzy number is used to modify the method of subtraction and division. A new technique is introduced by Das et al. [5] based on MOLP problems and Lexico-graphic method to solve FFLP with trapezoidal fuzzy numbers. Kiruthiga and Loganathan [7] presented a new way where fuzzy MOLP is reduced to crisp MOLP problems using ranking function and then crisp problem is solved by fuzzy programming method. Pandian [8] used a new method named level-sum method for finding an optimal fuzzy solution to a FLPP where fuzzy ranking function is not used. Karthy and Ganesan [9] describes fuzzy optimal compromise solution which is acquired by using Fuzzy Genetic Algorithm. In Zadeh [11], the primary work can be found on the weighted sum method. The weighted sum method was applied by Kaski [12] in structural optimization. The ε constraint method was developed by Marglin and the equality constraint method was developed by Lin [13]. In this paper, in order to solve fuzzy linear programming, it can be broken down into multi-objective linear programming for both triangular and trapezoidal fuzzy numbers. Then two different methods can be used, ranking function and weighted sum method, to prepare the single objective linear programming from multi-objective linear programming. It can be shown that, the approximation doesn't vary and it is same for both of the ranking function and weighted sum method.

2 Preliminaries

In this part, we have given some basic idea of fuzzy sets, triangular fuzzy number and trapezoidal fuzzy number, which are very necessary for this paper.

Definition 1. A fuzzy set $\tilde{B}$ is defined by $\tilde{B} = \{(x, \mu_B(x)) : x \in B, \mu_B(x) \in [0,1]\}$. In the pair $(x, \mu_B(x))$, the first component x belong to the classical set B, the second component $\mu_B(x)$ belong to the interval [0,1] , called membership function **[10]**.

Definition 2. A fuzzy number $\tilde{B}$ is a triangular fuzzy number denoted by $\tilde{B} = (b_1, b_2, b_3)$ where b_1, b_2, b_3 are real numbers and its membership function is given by **[3]**:

$$\mu_{\tilde{B}}(x) = \begin{cases} \dfrac{x - b_1}{b_2 - b_1}, & b_1 \le x \le b_2 \\ \dfrac{x - b_2}{b_2 - b_3}, & b_2 \le x \le b_3 \\ 0, & otherwise \end{cases}$$

Definition 3. A triangular fuzzy number (b_1, b_2, b_3) is said to be non negative fuzzy number iff $b_1 \ge 0$ **[3].**

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Definition 4. Two triangular fuzzy number $\tilde{A}=(a_1,a_2,a_3)$ and $\tilde{B}=(b_1,b_2,b_3)$ are said to be equal if and only if $a_1=b_1, a_2=b_2, a_3=b_3$ **[3].**

Definition 5. A ranking function is a function $\Re: F(R)\to R$ which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{B}=(b_1,b_2,b_3)$ is a triangular fuzzy number then $\Re(\tilde{B})=\dfrac{b_1+2b_2+b_3}{4}$ **[3].**

Definition 6. The arithmetic operations on two fuzzy numbers $\tilde{A}=(a_1,a_2,a_3)$ and $\tilde{B}=(b_1,b_2,b_3)$ are given by **[3], [9]**:

(i) $\tilde{A}\oplus\tilde{B}=(a_1+b_1,a_2+b_2,a_3+b_3)$

(ii) $\tilde{A}-\tilde{B}=(a_1-b_1,a_2-b_2,a_3-b_3)$

(iii) $-\tilde{A}=-(a_1,a_2,a_3)=(-a_3,-a_2,-a_1)$

(iv) For the multiplication if $\tilde{A}=(a_1,a_2,a_3)$ be any triangular fuzzy number and $\tilde{B}=(b_1,b_2,b_3)$ be a non negative triangular fuzzy number then

$$\tilde{A}\otimes\tilde{B}=\begin{cases}(a_1b_1,a_2b_2,a_3b_3), & a_1\geq 0\\ (a_1b_3,a_2b_2,a_3b_3), & a_1<0, a_3\geq 0\\ (a_1b_1,a_2b_2,a_3b_3), & a_3<0\end{cases}$$

$$(v)\ \tilde{A}/\tilde{B}=\min(a_1/b_1,a_1/b_3,a_3/b_1,a_3/b_3), a_2/b_2, \max(a_1/b_1,a_1/b_3,a_3/b_1,a_3/b_3)$$

Definition 6. A fuzzy number $\tilde{B}=(b_1,b_2,b_3,b_4)$ is called a trapezoidal fuzzy number if its membership function is defined as follows **[5]**:

$$\mu_{\tilde{B}}(x)=\begin{cases}\dfrac{x-b_1}{b_2-b_1}, & b_1\leq x\leq b_2\\ 1, & b_2\leq x\leq b_3\\ \dfrac{x-b_4}{b_3-b_4}, & b_3\leq x\leq b_4\\ 0, & else\end{cases}$$

Definition 8. Two trapezoidal fuzzy numbers $\tilde{A}=(a_1,a_2,a_3,a_4)$ and $\tilde{B}=(b_1,b_2,b_3,b_4)$ are said to be equal if and only if $a_1=b_1, a_2=b_2, a_3=b_3, a_4=b_4$ **[5].**

Definition 9. A ranking function is a function $\Re: F(R)\to R$, where $F(R)$ is a set of fuzzy number defined on set of real numbers, which maps each fuzzy number into real line, where a natural order exists. Let $\tilde{B}=(b_1,b_2,b_3,b_4)$ is a trapezoidal fuzzy number then $\Re(\tilde{B})=\dfrac{b_1+b_2+b_3+b_4}{4}$ **[5].**

Definition 10. The arithmetic operations on two non-negative trapezoidal fuzzy numbers $\tilde{A}=(a_1,a_2,a_3,a_4)$ and $\tilde{B}=(b_1,b_2,b_3,b_4)$ are given by **[5]**:

(i) $\tilde{A}\oplus\tilde{B}=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4)$

(ii) $\tilde{A}-\tilde{B}=(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1)$

(iii) $\tilde{A}\otimes\tilde{B}=(\alpha,\beta,\gamma,\delta)$

Where

$$\alpha=\min(a_1b_1,\ a_1b_4, a_4b_1, a_4b_4)$$
$$\beta=\min(a_2b_2,\ a_2b_3, a_3b_2, a_3b_3)$$
$$\gamma=\max(a_2b_2,\ a_2b_3, a_3b_2, a_3b_3)$$
$$\delta=\max(a_1b_1,\ a_1b_4, a_4b_1, a_4b_4)$$

3 METHODOLOGY

3.1 Multi-Objective Linear Programming [7]

A multi-objective linear programming (MOLP) can be stated as follows:

$$\text{Maximize } f_1(x),\ldots\ldots, f_k(x)$$

Subject to $x\in X=\{x\in R^n / g_j(x)\leq 0, j=1,2,\ldots,m$

Where $f_i, i=1,\ldots\ldots,k$ are the k distinct linear objective function of the decision variables and X is the feasible set of constrained decisions.

3.2 Fully Fuzzy Linear Programming Problem [3, 8]

Consider the following linear programming problem:

$$\text{Maximize } \tilde{z}=\tilde{r}^t\,\tilde{y}$$
$$\text{subject to } \tilde{A}\,\tilde{y}\,\{\leq=\geq\}\,\tilde{b}$$

$\tilde{y}$ is a non negative fuzzy number. Where

$\tilde{r}^t=[\tilde{r}_j]_{1\times n}$, $\tilde{y}=[\tilde{y}_j]_{n\times 1}$, $\tilde{A}=[\tilde{a}_{ij}]_{m\times n}$, $\tilde{b}=[\tilde{b}_i]_{m\times 1}$ and $\tilde{a}_{ij},\tilde{r}_j,\tilde{y}_j,\tilde{b}_i\in F(R)$ is called fully fuzzy linear programming problem (FFLP), where m is fuzzy equality constraints and n is fuzzy variables.

Let the parameters $\tilde{z},\tilde{a}_{ij},\tilde{r}_j,\tilde{y}_j$ and $\tilde{b}_i$ be the triangular fuzzy number $(z_1,z_2,z_3),(a_{ij},b_{ij},c_{ij}),(d_j,e_j,f_j),(y_j,x_j,t_j)$ and (b_i,p_i,q_i) respectively.

Then the program can be written as follows:

$$\text{Maximize}(z_1,z_2,z_3)=\sum_{j=1}^{n}(d_j,e_j,f_j)\otimes(y_j,x_j,t_j)$$

Subject to

$$\sum_{j=1}^{n}(a_{ij},b_{ij},c_{ij})\otimes(y_j,x_j,t_j)\{\leq=\geq\}(b_i,p_i,q_i)$$

Consider FFLP as a MOLP problem:

Maximize $z_1=\sum_{j=1}^{n}$ lower value of $(d_j,e_j,f_j)\otimes(y_j,x_j,t_j)$

Maximize $z_2=\sum_{j=1}^{n}$ middle value of $(d_j,e_j,f_j)\otimes(y_j,x_j,t_j)$

Maximize $z_3=\sum_{j=1}^{n}$ upper value of $(d_j,e_j,f_j)\otimes(y_j,x_j,t_j)$

Subject to

$$\sum_{j=1}^{n}\text{lower valueof }(a_{ij},b_{ij},c_{ij})\otimes(y_j,x_j,t_j)\{\leq=\geq\}b_i$$

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$$\text{for all } i = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} \text{ middle value of } (a_{ij}, b_{ij}, c_{ij}) \otimes (y_j, x_j, t_j) \{\le = \ge\}\ p_i$$

$$\text{for all } i = 1, 2, \ldots, n$$

$$\sum_{j=1}^{n} \text{ lower value of } (a_{ij}, b_{ij}, c_{ij}) \otimes (y_j, x_j, t_j) \{\le = \ge\} q_i$$

$$\text{for all } i = 1, 2, \ldots, n$$

$$z_2 \ge z_1,\ z_3 \ge z_2,\ y_j \le x_j,\ \ x_j \le t_j,\ y_j \ge 0,\ j = 1, 2, \ldots m$$

Remarks: In the case of a FFLP problem involving trapezoidal fuzzy numbers, we get a MOLP Problem having four objectives.

3.3 Multi-objective Fuzzy Linear Programming Problem [6]

The general form of Fuzzy multi-objective linear programming problem is given below:

$$\text{Maximize} \quad \tilde{z}^1, \tilde{z}^2, \ldots, \tilde{z}^k$$

Where $\tilde{z}^k = \sum_{j=1}^{n} \tilde{r}_j^k y_j$, $k = 1, 2, \ldots, k$

Subject to $\sum_{j=1}^{n} \tilde{a}_{ij} y_j \le \tilde{b}_i$, $i = 1, 2, \ldots, m$

$$y_j \ge 0,\ j = 1, 2, \ldots, n$$

Where $\tilde{r}_j, \tilde{a}_{ij}$ and $\tilde{b}_i$ are fuzzy numbers and y_j are fuzzy variables whose states are fuzzy numbers.

4 Adaptive Weighted Sum Method to find Single Objective from MOLP

In 1970s, in the field of engineering and science the conception of pareto optimality was linked by Stadler [14, 15]. A mathematical model is presented by Alanazi, H. O et al. [4]. For MOLP, the most broadly used method is the weighted sum method. By multiplying each objective function by a weighting factor this method changes the multiple objectives into a single objective function with the help of summing up all weighted objective functions.

$f_{weighted\,sum} = \alpha_1 f_1 + \alpha_2 f_2 + \ldots\ldots + \alpha_s f_s$ where α_i is a weighting factor for the *i*-th objective function. The weighted sum is said to be a convex combination of objectives if

$$\sum_{i=1}^{s} \alpha_i = 1 \quad and\ 0 \le \alpha_i \le 1.$$

One particular optimal solution point is determined by each objective optimization on the pareto front. Each different single objective optimization determines a different optimal solution and then the weighted sum method changes weights systematically.

Kim [16] represents an adaptive weighted sum method for MOLP problems. In a general form, the weighted single objective of *m* objective functions J_{Total}^{s} is defined by

$$J_{Total}^{s} = \alpha_{s-1} J_{Total}^{s-1} + (1 - \alpha_{s-1}) J_s, \quad s \ge 2$$

where $J_{Total}^{1} = J_1$. Note that (*s*-1) weighting factors are needed to explore *s*- dimensional objective space and α_i is the *i*-th weighting factor.

Consider the single objective function

$$J_{Total}^{s} = \alpha_{s-1} J_{Total}^{s-1} + (1 - \alpha_{s-1}) J_s, \quad s \ge 2$$

For 2 objective functions, objective function is

$$J_T^2 = \alpha_1 J_T^1 + (1 - \alpha_1) J_2 = \alpha_1 J_1 + (1 - \alpha_1) J_2$$

For 3 objective functions, objective function is

$$\begin{aligned} J_T^3 &= \alpha_2 J_T^2 + (1 - \alpha_2) J_3 \\ &= \alpha_2 [\alpha_1 J_1 + (1 - \alpha_1) J_2] + (1 - \alpha_2) J_3 \\ &= \alpha_1 \alpha_2 J_1 + (1 - \alpha_1) \alpha_2 J_2 + (1 - \alpha_2) J_3 \end{aligned}$$

For 4 objective functions, objective function is

$$\begin{aligned} J_T^4 &= \alpha_3 J_T^3 + (1 - \alpha_3) J_4 \\ &= \alpha_3 [\alpha_1 \alpha_2 J_1 + (1 - \alpha_1) \alpha_2 J_2 + (1 - \alpha_2) J_3] + (1 - \alpha_3) J_4 \end{aligned}$$

5 NUMERICAL EXAMPLE

5.1 Triangular fuzzy number:

Consider the following fuzzy linear programming problem

$$Maz\ z = (1, 6, 9) \otimes \tilde{x}_1 \oplus (2, 3, 8) \otimes \tilde{x}_2$$

Subject to

$$(2, 3, 4) \otimes (x_1, y_1, z_1) \oplus (1, 2, 3) \otimes (x_2, y_2, z_2) = (6, 16, 30)$$
$$(-1, 1, 2) \otimes (x_1, y_1, z_1) \oplus (1, 3, 4) \otimes (x_2, y_2, z_2) = (1, 17, 30)$$

$$Max\ z = (x_1 + 2x_2,\ 6y_1 + 3y_2,\ 9z_1 + 8z_2)$$

Subject to

$$(2x_1 + x_2,\ 3y_1 + 2y_2,\ 4z_1 + 3z_2) = (6, 16, 30)$$
$$(-x_1 + x_2,\ y_1 + 3y_2,\ 2z_1 + 4z_2) = (1, 17, 30)$$

Using Ranking function,

$$Max\ z = \frac{1}{4} x_1 + \frac{1}{2} x_2 + 3y_1 + \frac{3}{2} y_2 + \frac{9}{4} z_1 + 2z_2$$

Subject to,

$$\begin{aligned} 2x_1 + x_2 &= 6 \\ -x_1 + x_2 &= 1 \\ 3y_1 + 2y_2 &= 16 \\ y_1 + 3y_2 &= 17 \\ 4z_1 + 3z_2 &= 30 \\ 2z_1 + 4z_2 &= 30 \end{aligned}$$

The optimal solution is

$$(x_1 = 1.7, y_1 = 2, z_1 = 3), (x_2 = 2.7, y_2 = 5, z_2 = 6)$$

Using weight function,

$$\begin{aligned} Max\ z &= \frac{1}{3}(x_1 + 2x_2 + 6y_1 + 3y_2 + 9z_1 + 8z_2) \\ &= \frac{1}{3} x_1 + \frac{2}{3} x_2 + 2y_1 + y_2 + 3z_1 + \frac{8}{3} z_2 \end{aligned}$$

Subject to

$$\begin{aligned} 2x_1 + x_2 &= 6 \\ -x_1 + x_2 &= 1 \\ 3y_1 + 2y_2 &= 16 \\ y_1 + 3y_2 &= 17 \\ 4z_1 + 3z_2 &= 30 \\ 2z_1 + 4z_2 &= 30 \end{aligned}$$

For equal weight the optimal solution is

$$(x_1 = 1.7, y_1 = 2, z_1 = 3), (x_2 = 2.7, y_2 = 5, z_2 = 6)$$

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$$Max\ z = 0.105(x_1 + 2x_2) + 0.195(6y_1 + 3y_2) + 0.7(9z_1 + 8z_2)$$
$$= x_1(0.105) + x_2(2\times 0.105) y_1(6\times 0.195) + y_2(3\times 0.195)$$
$$+ z_1(9\times 0.7) + z_2(8\times 0.7)$$
$$= x_1(0.105) + x_2(0.21) y_1(1.17) + y_2(0.585) + z_1(6.3) + z_2(5.6)$$

Subject to

$$2x_1 + x_2 = 6$$
$$-x_1 + x_2 = 1$$
$$3y_1 + 2y_2 = 16$$
$$y_1 + 3y_2 = 17$$
$$4z_1 + 3z_2 = 30$$
$$2z_1 + 4z_2 = 30$$

For unequal weight the optimal solution is

$$(x_1 = 1.7, y_1 = 2, z_1 = 3), (x_2 = 2.7, y_2 = 5, z_2 = 6)$$

5.2 Trapezoidal fuzzy number:

Consider the following fuzzy linear programming problem

$$Maz\ z = (1, 2, 3, 4) \otimes \tilde{x}_1 \oplus (2, 3, 4, 5) \otimes \tilde{x}_2$$

Subject to

$$(0, 1, 2, 3) \otimes (x_1, y_1, z_1, u_1) \oplus (1, 2, 3, 4) \otimes (x_2, y_2, z_2, u_2) = (2, 10, 24, 5)$$
$$(1, 2, 3, 4) \otimes (x_1, y_1, z_1, u_1) \oplus (0, 1, 2, 3) \otimes (x_2, y_2, z_2) = (1, 8, 21, 7)$$

$$Max\ z = (x_1 + 2x_2, 2y_1 + 3y_2, 3z_1 + 4z_2, 4u_1 + 5u_2)$$

Subject to

$$(x_2,\ y_1 + 2y_2, 2z_1 + 3z_2, 3u_1 + 4u_2) = (2, 10, 24, 5)$$
$$(x_1,\ 2y_1 + y_2, 3z_1 + 2z_2, 4u_1 + 3u_2) = (1, 8, 21, 7)$$

Using ranking function

$$\frac{1}{4}(x_1 + 2x_2 + 2y_1 + 3y_2 + (x_1 + 2x_2 - 3z_1 - 4z_2) + (2y_1 + 3y_2 + 4u_1 + 5u_2))$$

$$Max\ z = \frac{1}{2}x_1 + x_2 + y_1 + \frac{3}{2}y_2 - \frac{3}{4}z_1 - z_2 + u_1 + \frac{5}{4}u_2$$

Subject to

$$x_1 = 1$$
$$x_2 = 2$$
$$2y_1 + y_2 = 8$$
$$y_1 + 2y_2 = 10$$
$$3z_1 + 2z_2 = 21$$
$$2z_1 + 3z_2 = 24$$
$$4u_1 + 3u_2 = 7$$
$$3u_1 + 4u_2 = 5$$

The optimal solution is

$$(x_1 = 1, y_1 = 2, z_1 = 3, u_1 = 1.7), (x_2 = 2, y_2 = 4, z_2 = 6, u_2 = 0)$$

Using weight function,

$$Max\ z = \frac{1}{4}(x_1 + 2x_2 + 2y_1 + 3y_2 + 3z_1 + 4z_2 + 4u_1 + 5u_2)$$
$$= \frac{1}{4}x_1 + \frac{1}{2}x_2 + \frac{1}{2}y_1 + \frac{3}{4}y_2 + \frac{3}{4}z_1 + z_2 + u_1 + \frac{5}{4}u_2$$

Subject to

$$x_1 = 1$$
$$x_2 = 2$$
$$2y_1 + y_2 = 8$$
$$y_1 + 2y_2 = 10$$
$$3z_1 + 2z_2 = 21$$
$$2z_1 + 3z_2 = 24$$
$$4u_1 + 3u_2 = 7$$
$$3u_1 + 4u_2 = 5$$

For equal weight the optimal solution is

$$(x_1 = 1, y_1 = 2, z_1 = 3, u_1 = 1.7), (x_2 = 2, y_2 = 4, z_2 = 6, u_2 = 0)$$

$$Max\ z = 0.036(x_1 + 2x_2) + 0.108(2y_1 + 3y_2) + 0.306(3z_1 + 4z_2) + 0.55(4u_1 + 5u_2)$$
$$= 0.036\,x_1 + 0.072\,x_2 + 0.216\,y_1 + 0.324\,y_2 + 0.918\,z_1 + 1.224\,z_2 + 2.2\,u_1 + 2.75\,u_2$$

Subject to

$$x_1 = 1$$
$$x_2 = 2$$
$$2y_1 + y_2 = 8$$
$$y_1 + 2y_2 = 10$$
$$3z_1 + 2z_2 = 21$$
$$2z_1 + 3z_2 = 24$$
$$4u_1 + 3u_2 = 7$$
$$3u_1 + 4u_2 = 5$$

For unequal weight the optimal solution is

$$(x_1 = 1, y_1 = 2, z_1 = 3, u_1 = 1.7), (x_2 = 2, y_2 = 4, z_2 = 6, u_2 = 0)$$

6 Conclusion

For triangular fuzzy number a FLPP is solved by using both weighted sum method and ranking function. In case of weighted sum method, both equal and unequal weights have been used here. It is seen that weighted sum method and ranking function give the same result. Same investigation has been done for trapezoidal fuzzy number.

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